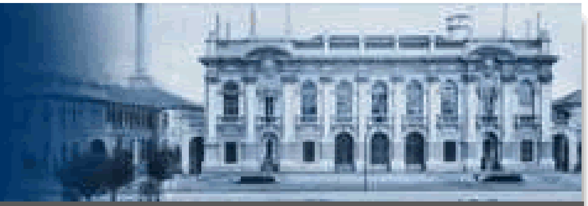
POLITECNICO DI MILANO
Piazza Leonardo da Vinci, 32 - 20133 Milano
Tel. +39.02.2399.1 - http://www.polimi.it



Advanced Course on

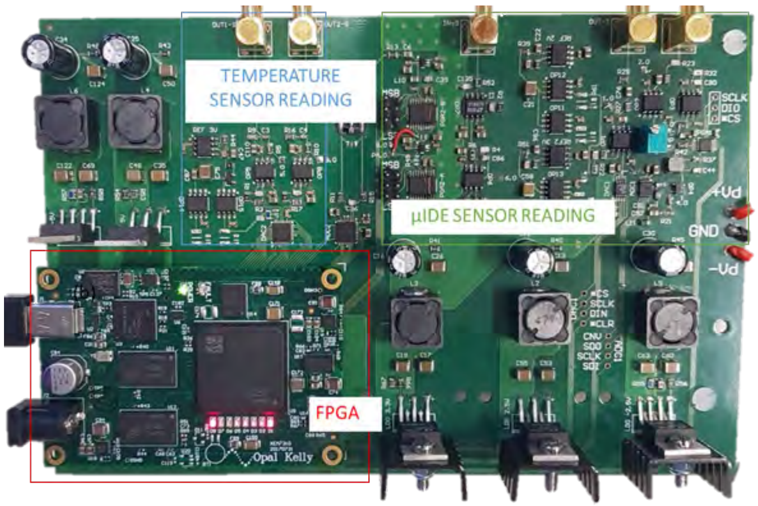
HIGH RESOLUTION ELECTRONIC MEASUREMENTS
IN NANO-BIO SCIENCE

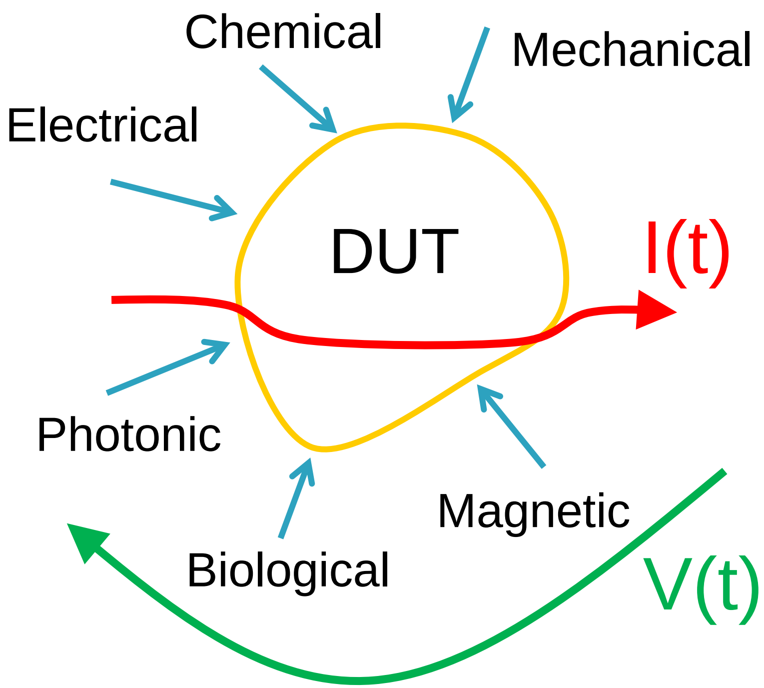
Impedance Measurement

Architectures and performance

Marco Sampietro

Why measuring impedance values

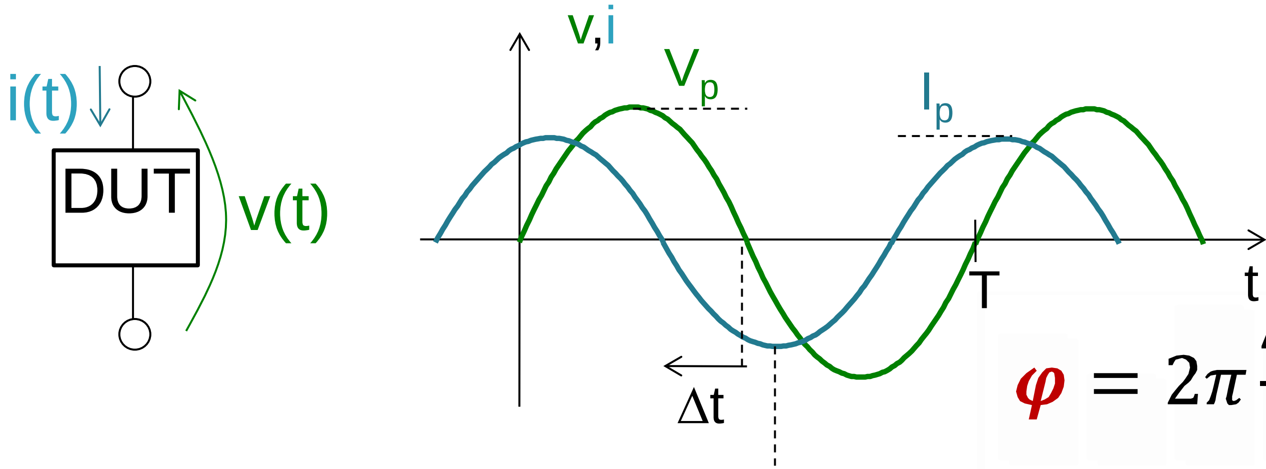




Extract R, L, C values in an electronic circuits …

… access the conduction properties and the dissipative properties of a new device/material/molecule/etc.

Definition



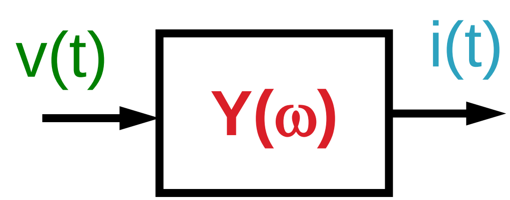
$$\varphi = 2\pi \frac{\Delta t}{T}$$

Amplitude
&
Phase
&
Frequency

$$Z = \frac{V}{I}$$ is a complex quantity

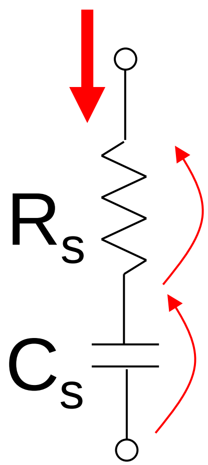
Impedance [Ohm]

Admittance [Siemens]



marco.sampietro@polimi.it

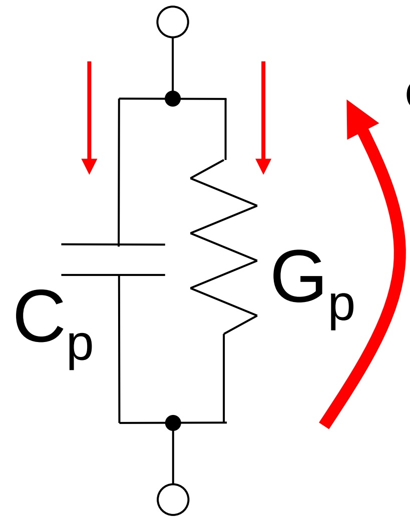
Impedance in terms of single R & C



Resistance   Reactance

$$Z(\omega) = R_s + \frac{1}{j\omega C_s}$$

Impedance



Conductance   Susceptance

$$Y(\omega) = G_p + j\omega C_p$$

Admittance

- Re{Z}, Re{Y} → energy dissipation (4kTRe{Z}, 4kTRe{Y})
- Im{Z}, Im{Y} → energy storage

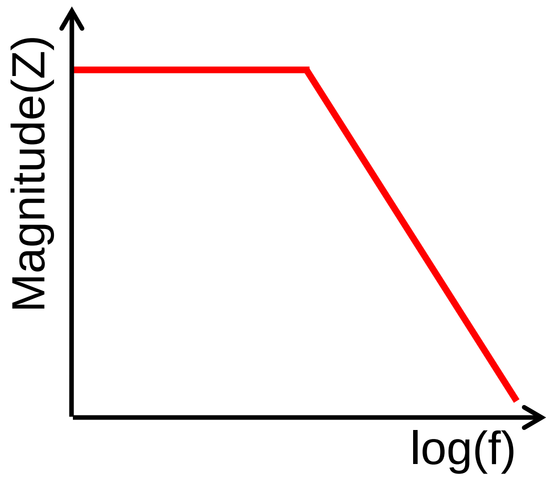
marco.sampietro@polimi.it

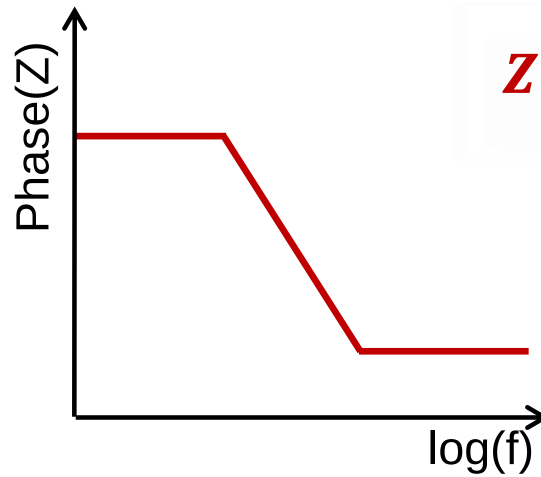
Plotting the Impedance

Equivalent ways to plot impedance / admittance values

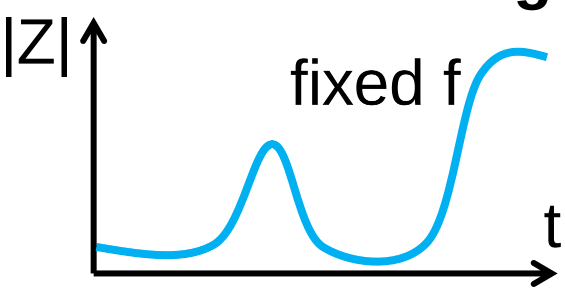
Bode Plots

$$Z(\omega) = R_s + \frac{1}{j\omega C_s}$$

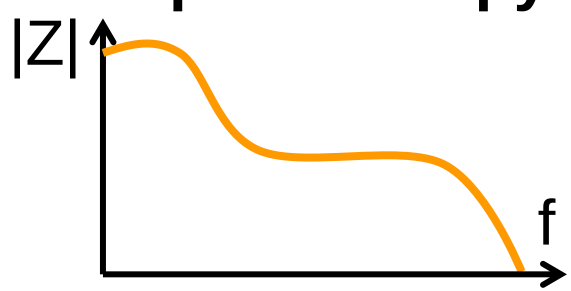




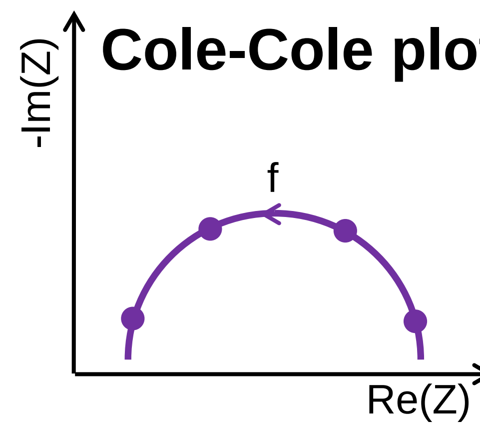
Time tracking



Spectroscopy



Cole-Cole plot



marco.sampietro@polimi.it

ChatGPT enquire

Myself : *How can I measure the impedance of a device ?*

chatGPT : Using an Impedance Analyzer

Using an LCR Meter

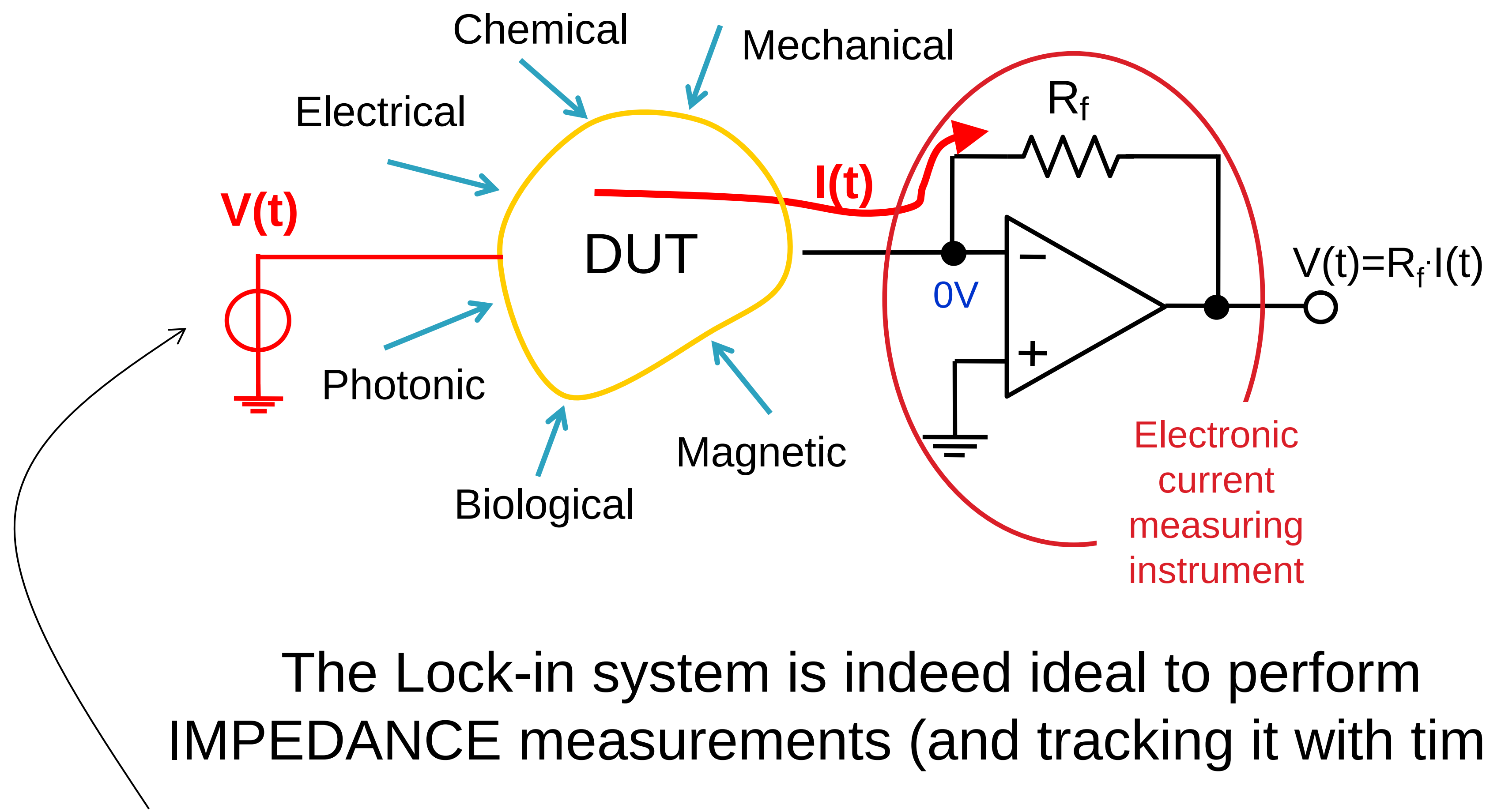
} Lock-in architecture

Bridge Circuit

Oscilloscope and Function Generator using the voltage-divider principle

marco.sampietro@polimi.it

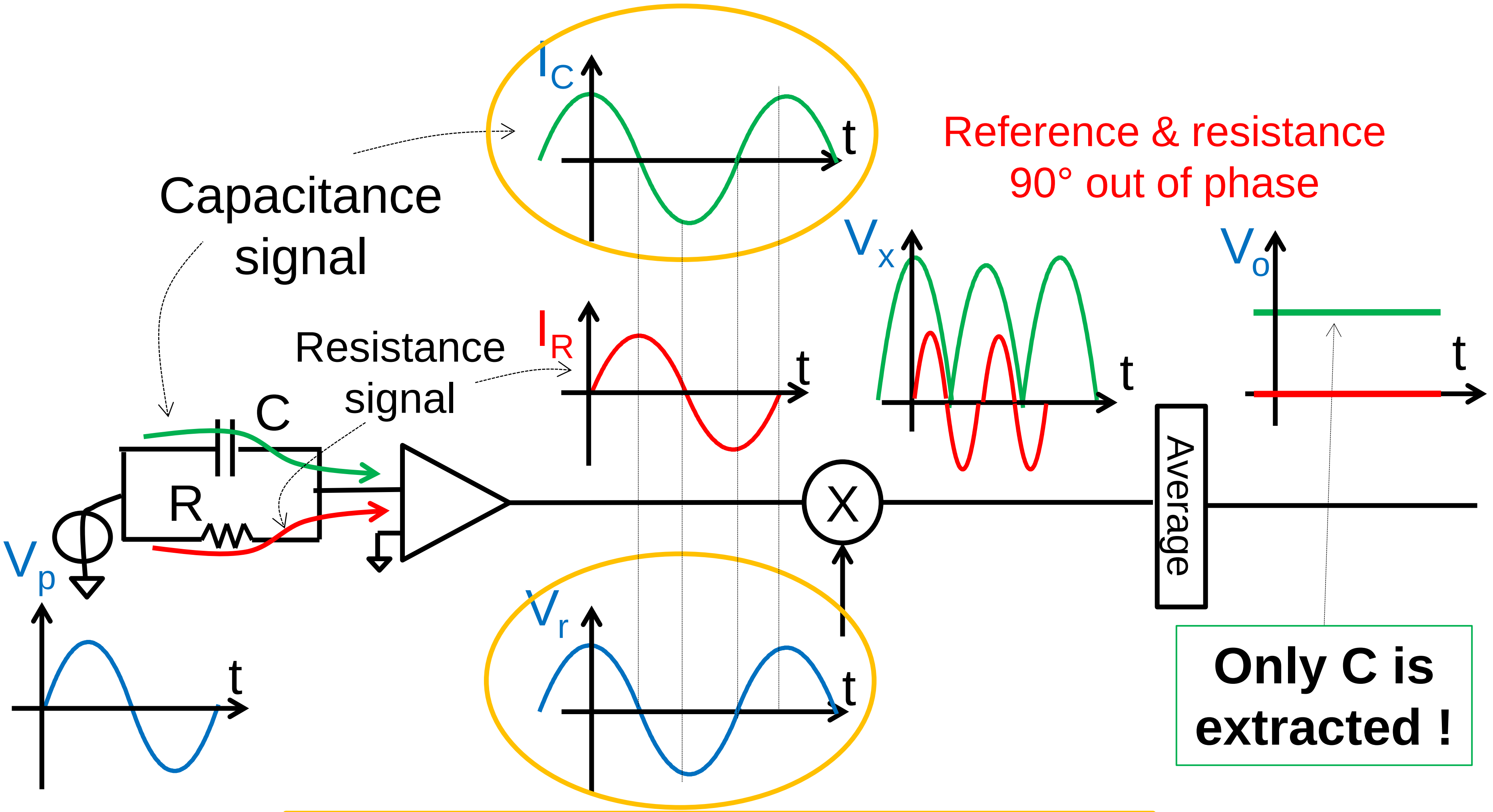
Lock-in configuration



The Lock-in system is indeed ideal to perform IMPEDANCE measurements (and tracking it with time)

By sweeping the frequency, you can easily perform IMPEDANCE SPECTRUM

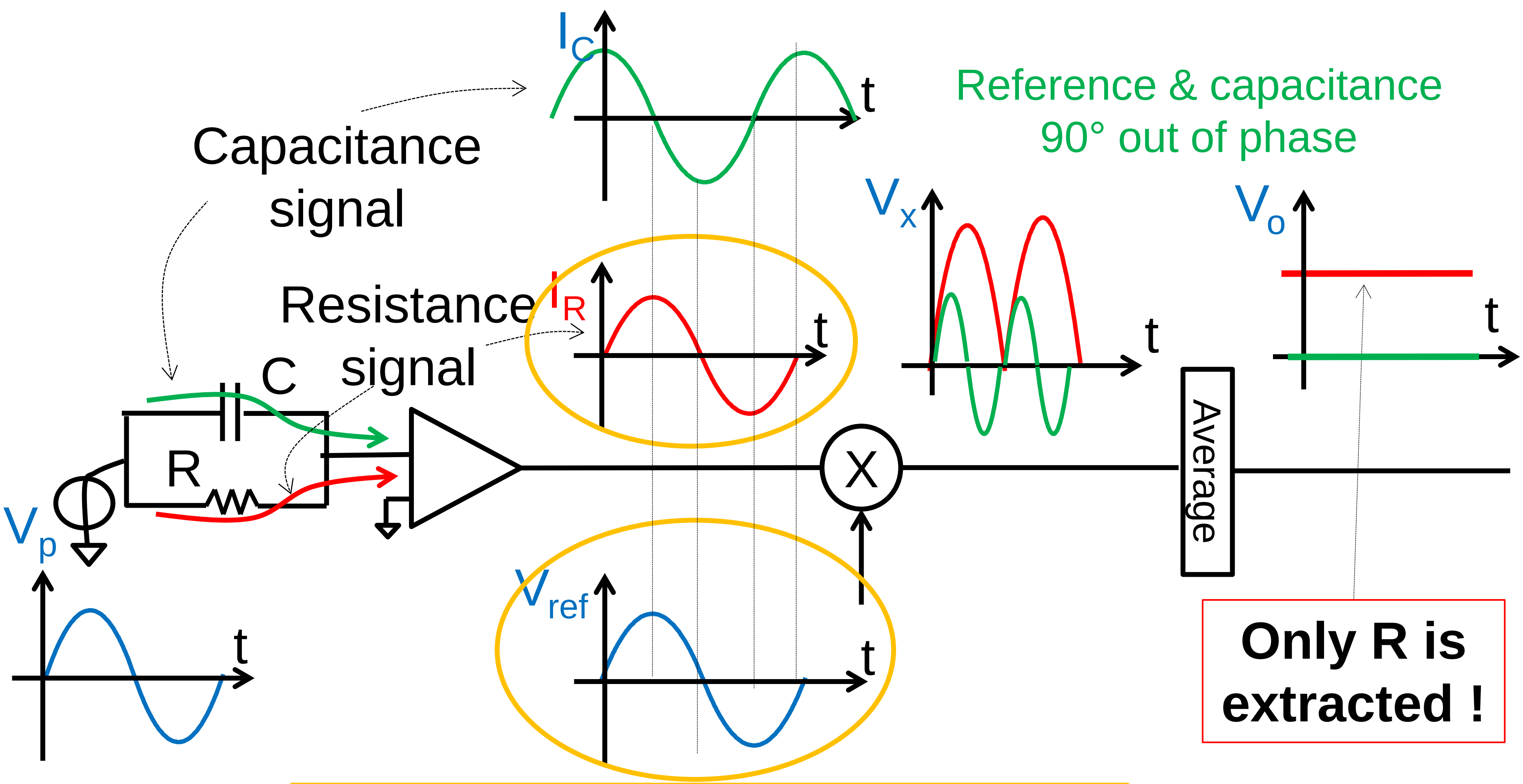
Mixture of R & C in real sensors



Only C is extracted !

Reference & capacitance : in phase

R & C selectivity of the LOCK-IN



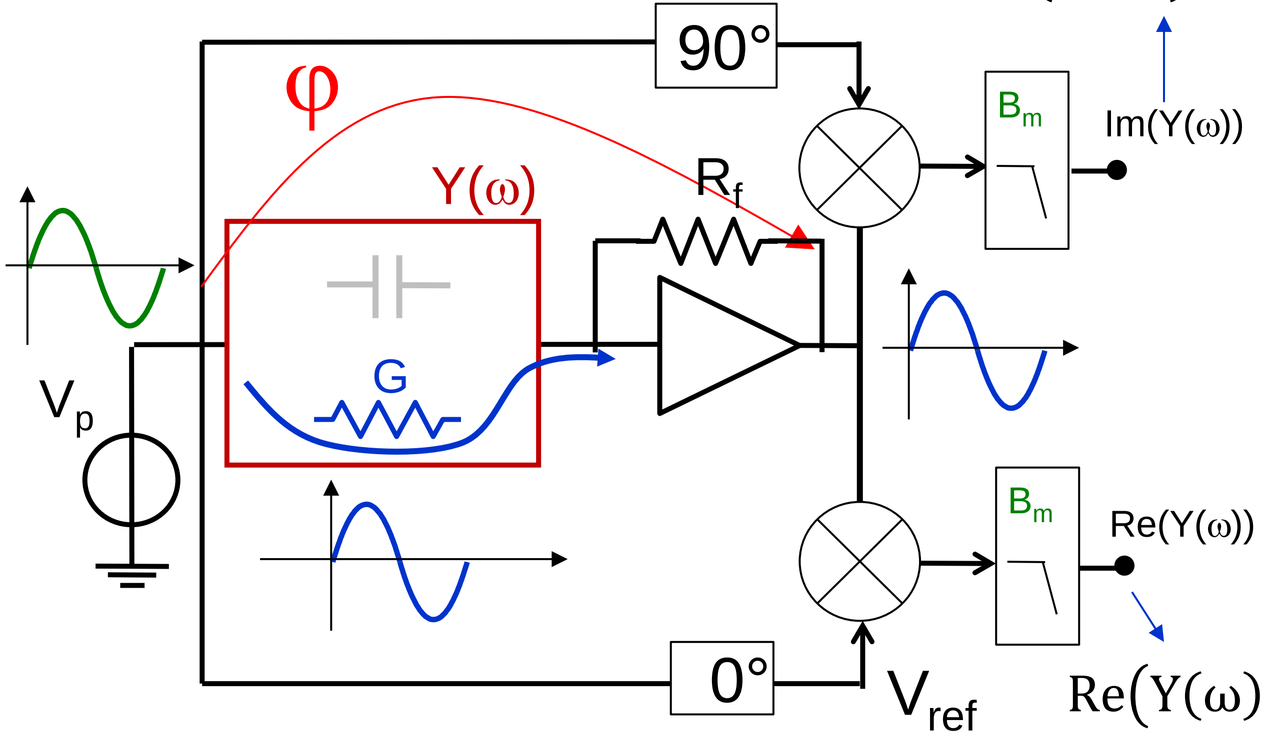
Only R is extracted !

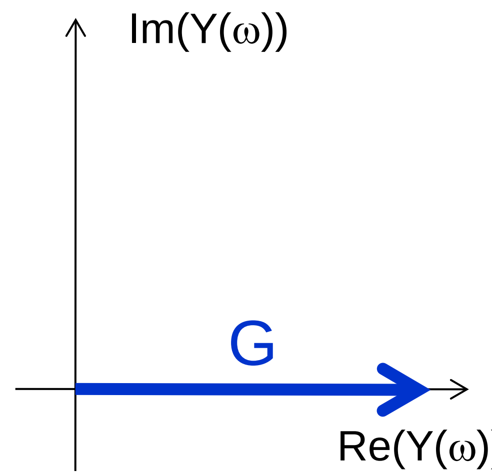
Reference & resistance : in phase

Double Lock-In: in-phase detection

Two multipliers are used to obtain both Re and Im of a DUT

$$\mathrm{Im}\big(Y(\omega)\big) = \tfrac{1}{2} R_f V_p V_{ref} |Y(\omega)| \sin(0°) = 0$$





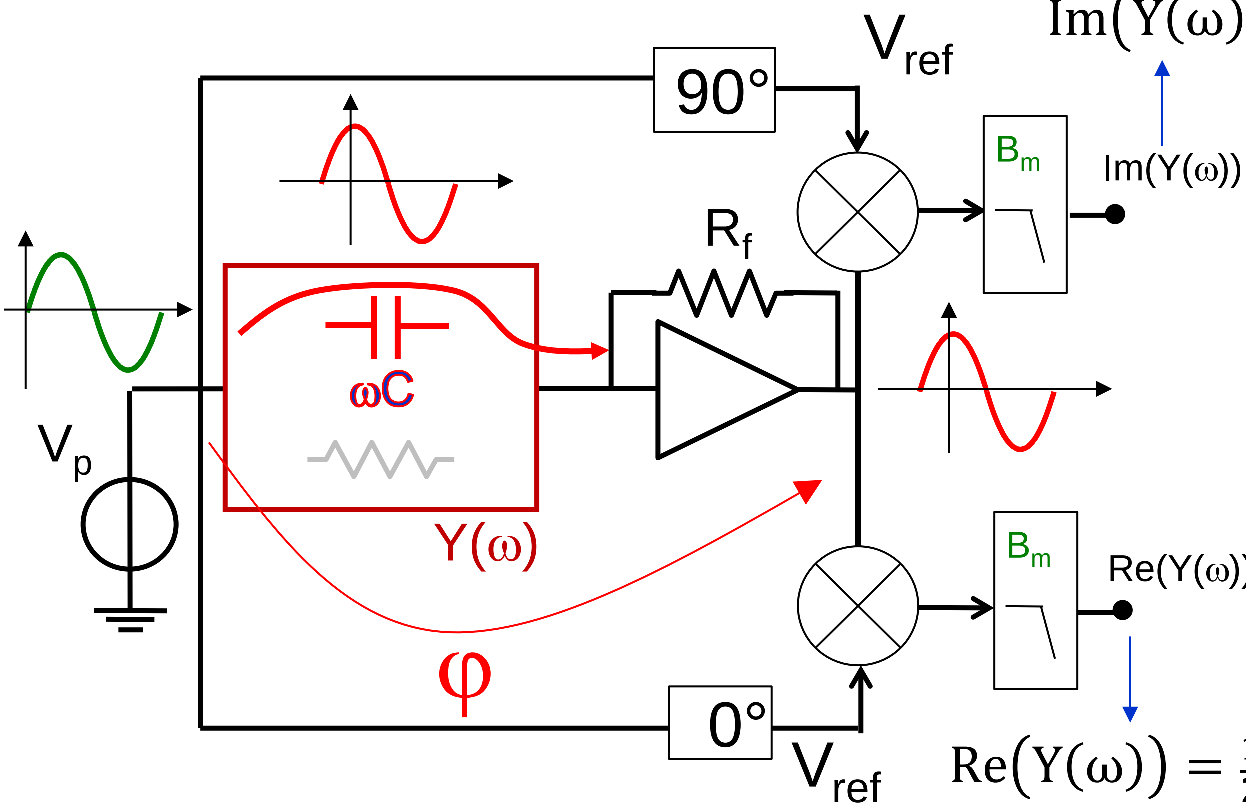
$$\mathrm{Re}\big(Y(\omega)\big) = \tfrac{1}{2} R_f V_p V_{ref} |Y(\omega)| \cos(0°)$$

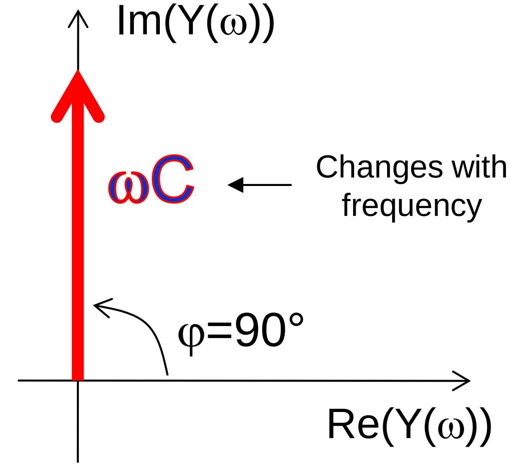
$$\mathbf{G} = \frac{2 \cdot \mathrm{Re}\big(Y(\omega)\big)}{R_f\, V_p V_{ref}}$$

Double Lock-In: in-quadrature detect.

$$\omega C = \frac{2 \cdot \mathrm{Im}(\mathrm{Y}(\omega))}{\mathrm{R_f} \mathrm{V}_p \mathrm{V}_{ref}}$$

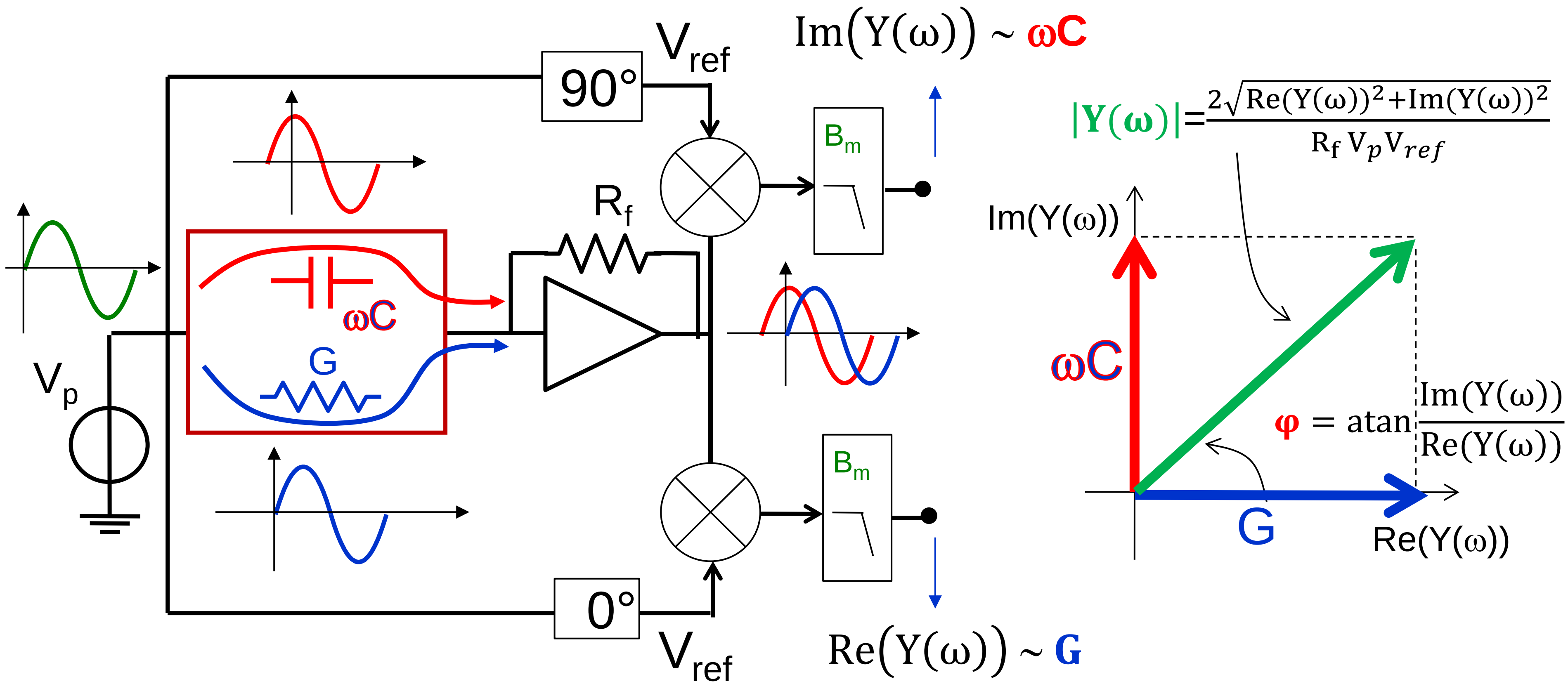
$$\mathrm{Im}(\mathrm{Y}(\omega)) = \tfrac{1}{2}\mathrm{R_f}\mathrm{V}_p\mathrm{V}_{ref}|\mathrm{Y}(\omega)|\sin(90°)$$





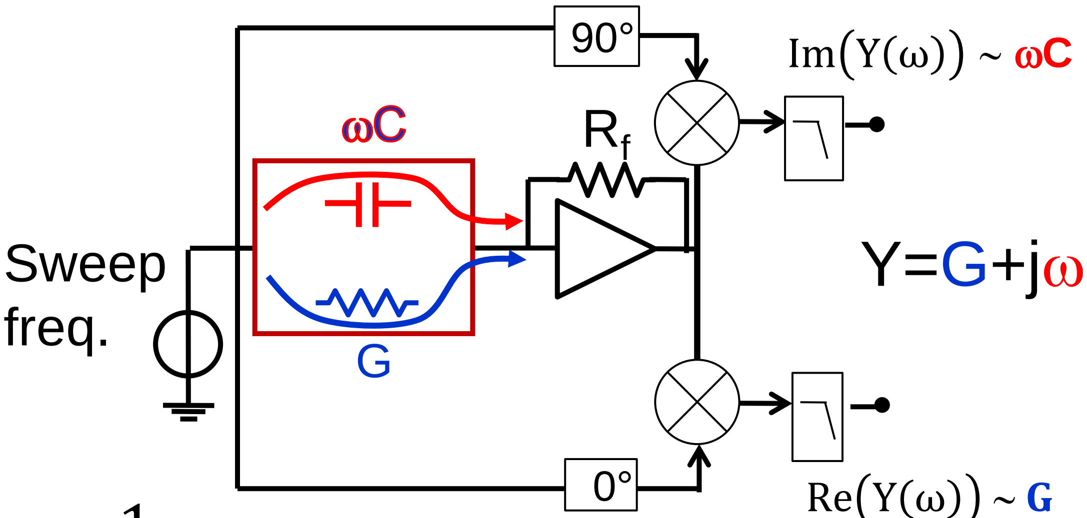
$$\mathrm{Re}(\mathrm{Y}(\omega)) = \tfrac{1}{2}\mathrm{R_f}\mathrm{V}_p\mathrm{V}_{ref}|\mathrm{Y}(\omega)|\cos(90°) = 0$$

Lock-In: Impedance of R||C



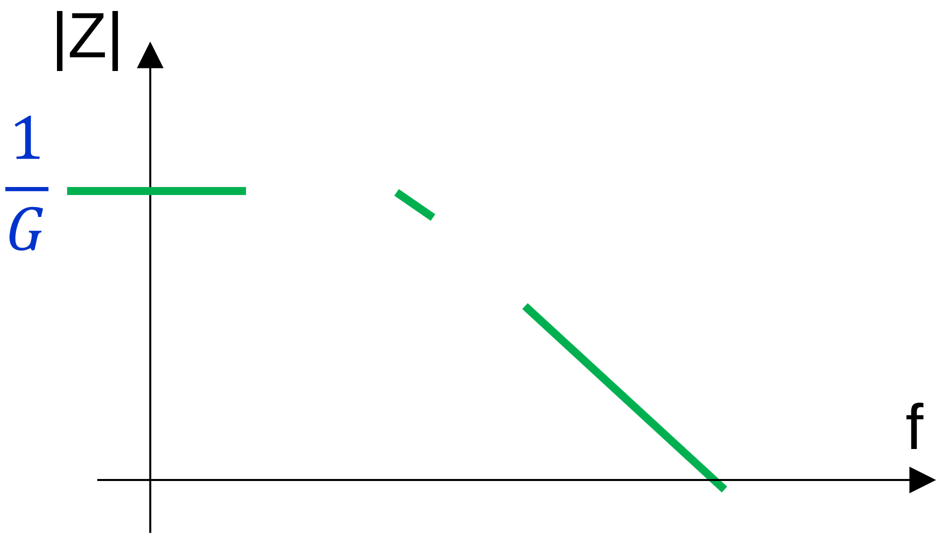
marco.sampietro@polimi.it

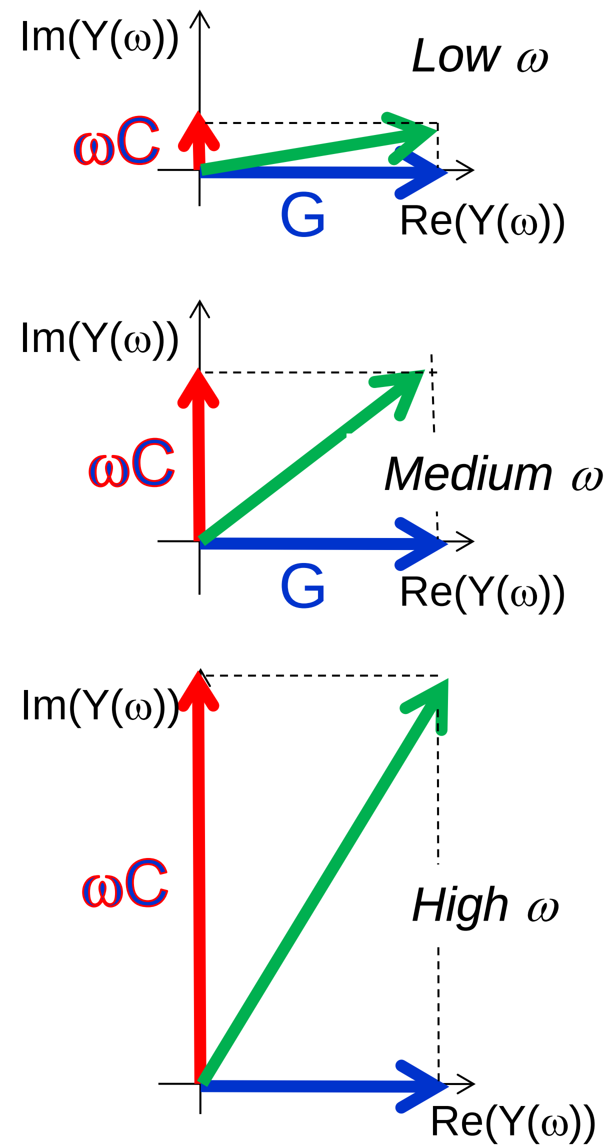
Sweeping the frequency : spectrum



Y=G+jωC

$$Z = \frac{1}{G}$$





marco.sampietro@polimi.it

Improving the resolution

You have a good set-up for the measurement of the capacitance of a DUT based on a lock-in platform. Using $V_p=V_{ref}=0.1V$ and by averaging the measurement for $T_m=30ms$, you obtain a resolution of $\Delta C=12aF$ in the sensor.

If a $\Delta C=1aF$ is required, what will you do?

Extend the measuring time

$$\Delta C \propto \frac{1}{\sqrt{T_m}}$$

$T_m=30ms$ ➡ $T_m=4.3s$

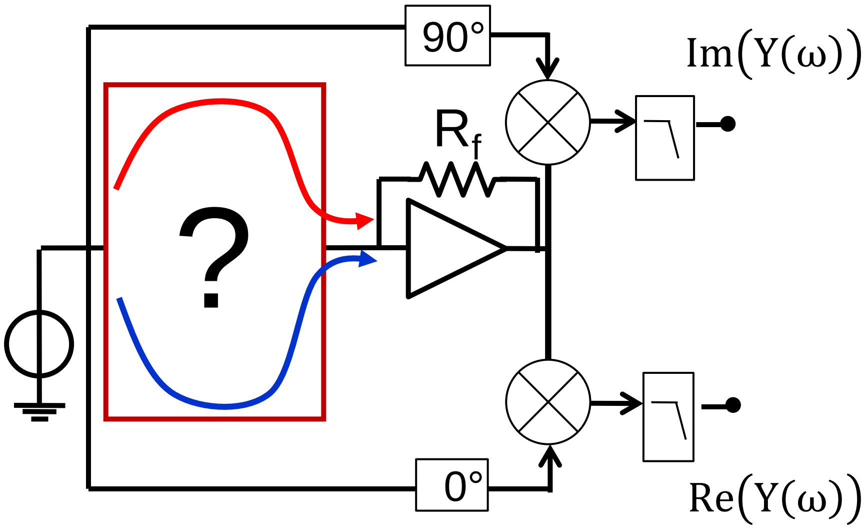
Increase the amplitude of V_p or V_{ref} or both

$$\Delta\omega C \propto \frac{1}{R_f V_p V_{ref}}$$

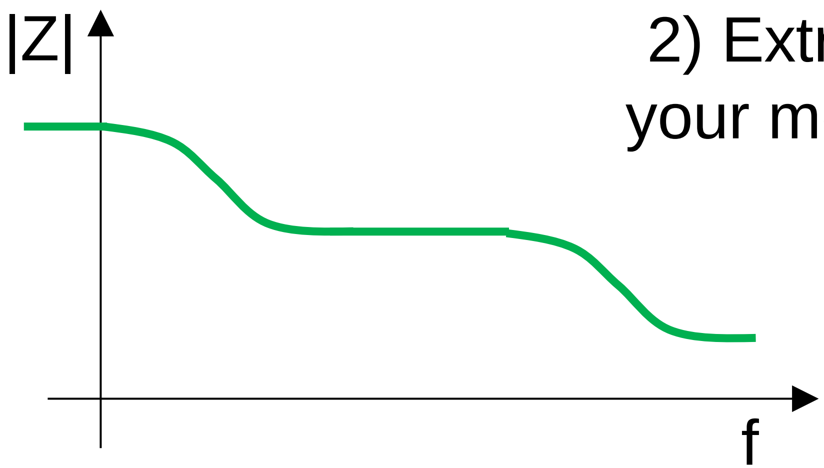
$V_{ref}=0.1V$ ➡ $V_{ref}=1.2V$

Extracting elements from a spectrum

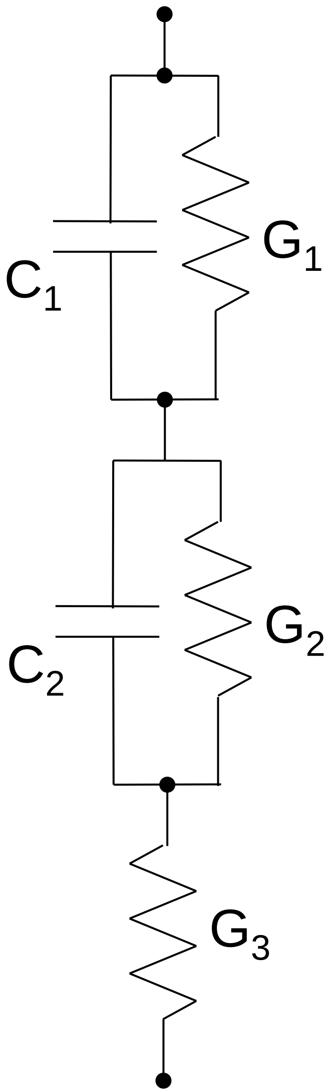
What is inside my device ?



1) Measure at different frequencies

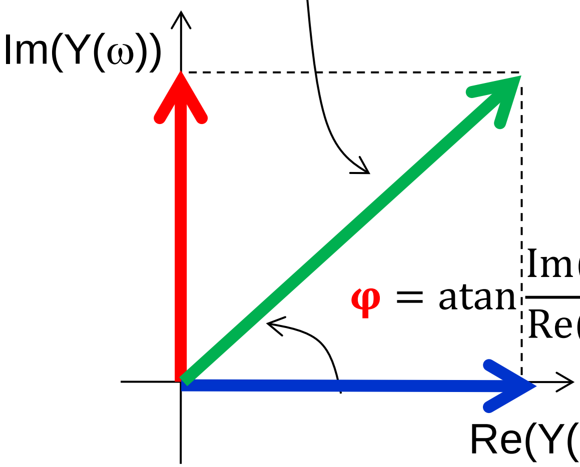


2) Extract your model



marco.sampietro@polimi.it

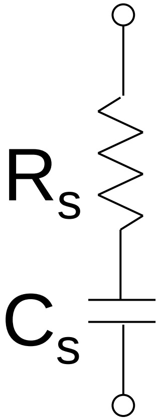
Pre-defined models in LCR meters



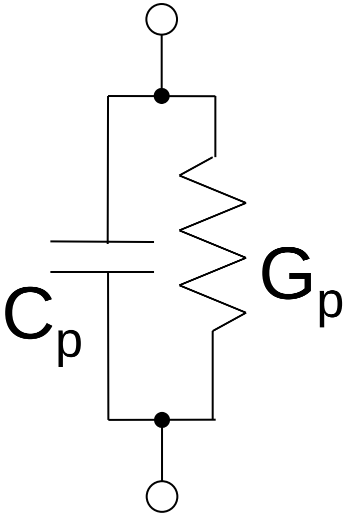
$$|Y(\omega)| = \frac{2\sqrt{Re(Y(\omega))^2 + Im(Y(\omega))^2}}{R_f V_p V_{ref}}$$

$$\varphi = \operatorname{atan}\frac{Im(Y(\omega))}{Re(Y(\omega))}$$

You select the model.
The instrument gives you the R & C values



$$Z(\omega) = R_s + \frac{1}{j\omega C_s}$$



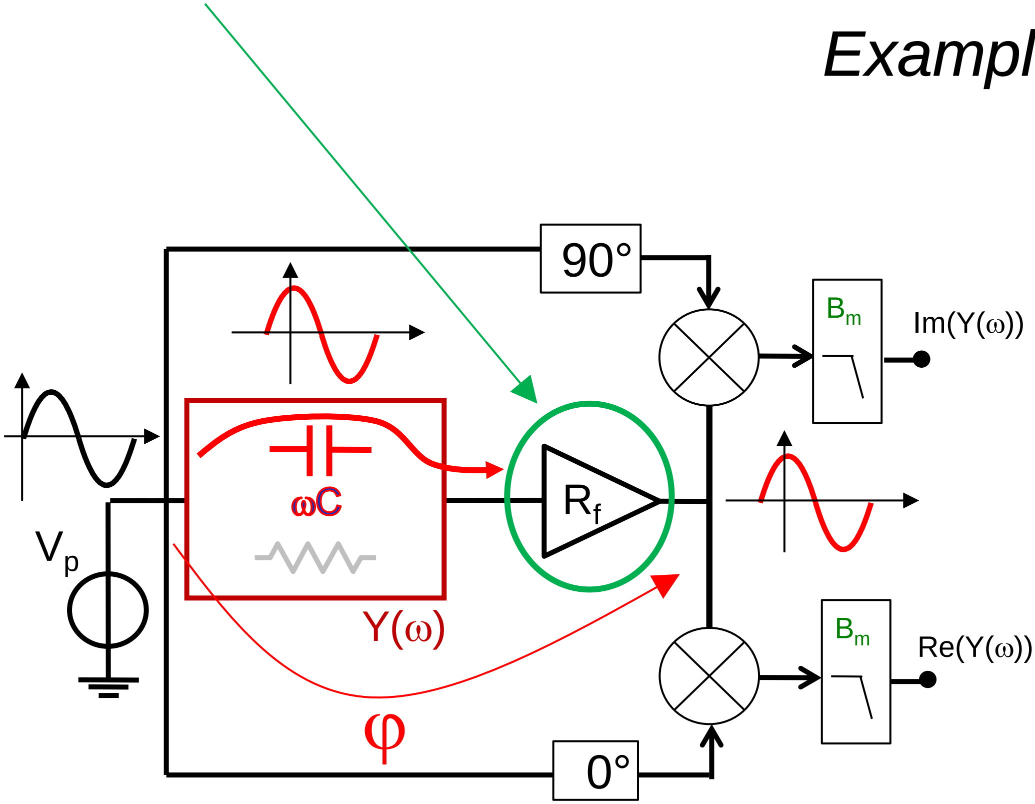
$$Y(\omega) = G_p + j\omega C_p$$

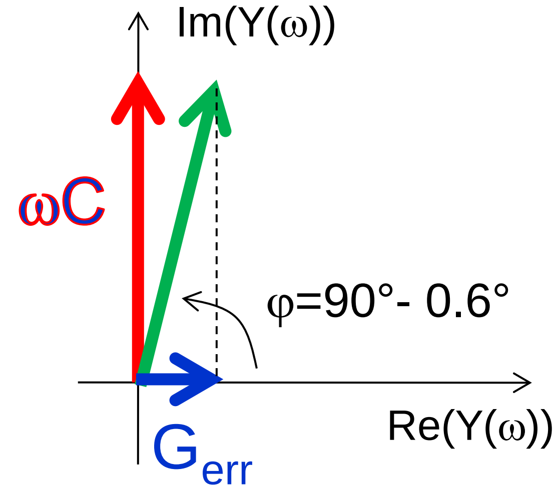
marco.sampietro@polimi.it

Calibration

Amplifiers and connections introduce errors in amplitude and phase

Example: C=1pF at 1MHz
Phase error 0.6°
(a pole distant two decades)



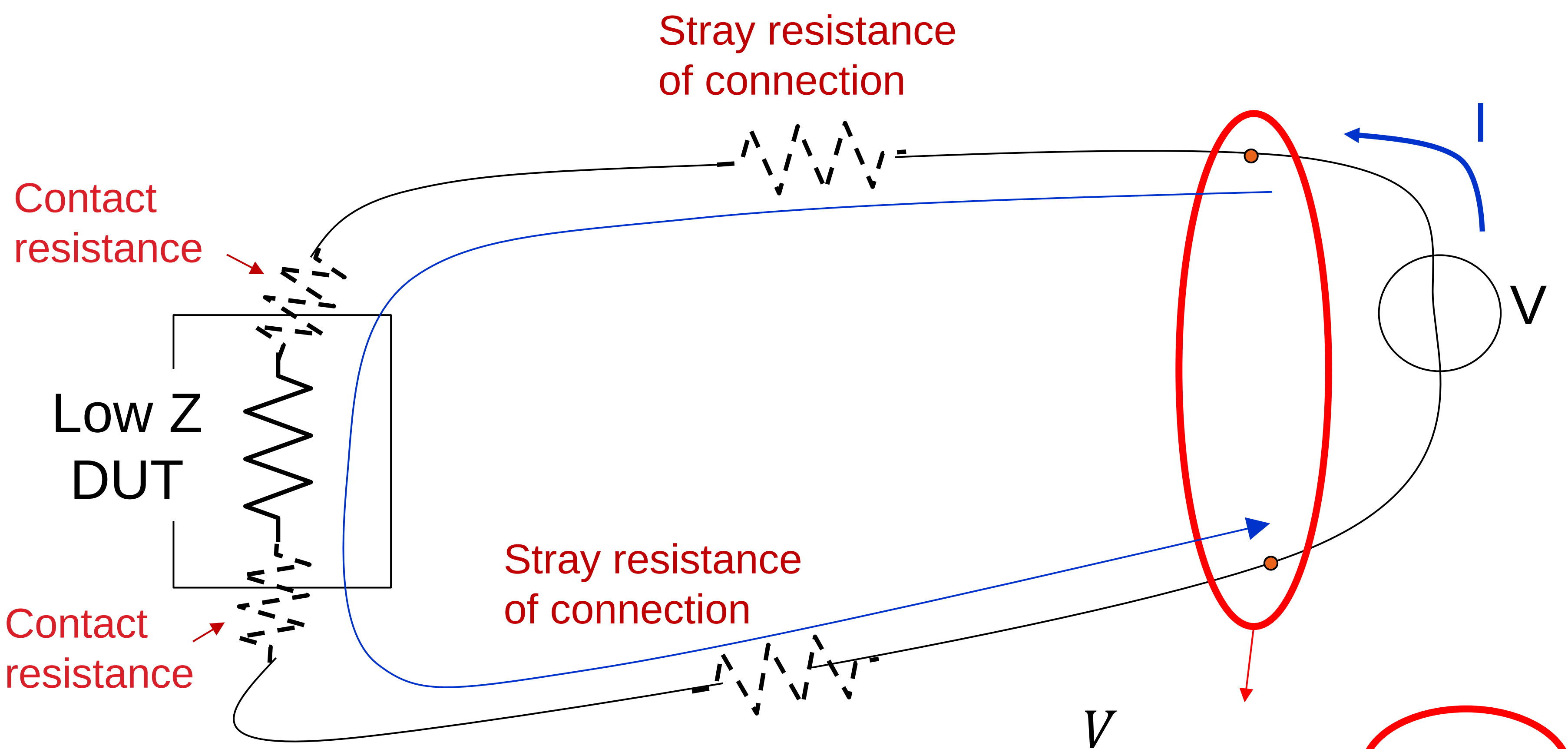


$G_{err} = \omega C_x \sin(0.6°) = 6 \cdot 10^{-8}$ S
(16MΩ, to be compared with ∞)

If φ_{err}=10° than $1/G_{err}$ =1MΩ !

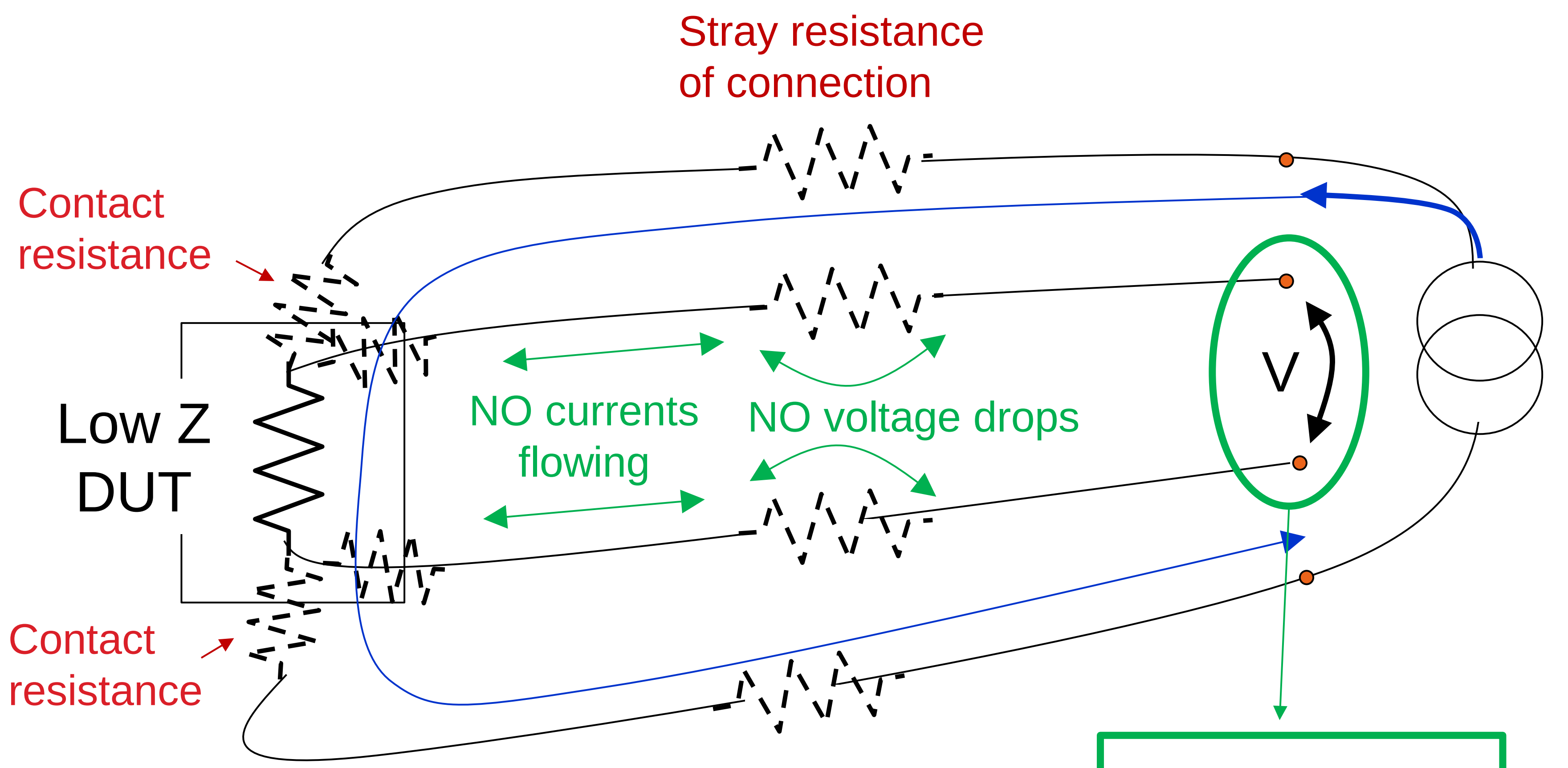
Ideal calibration: with a known sample (amplitude and phase)

Contribution of strays (resistances)



$$Z = \frac{V}{I} = Z_{DUT} + R_{strays}$$

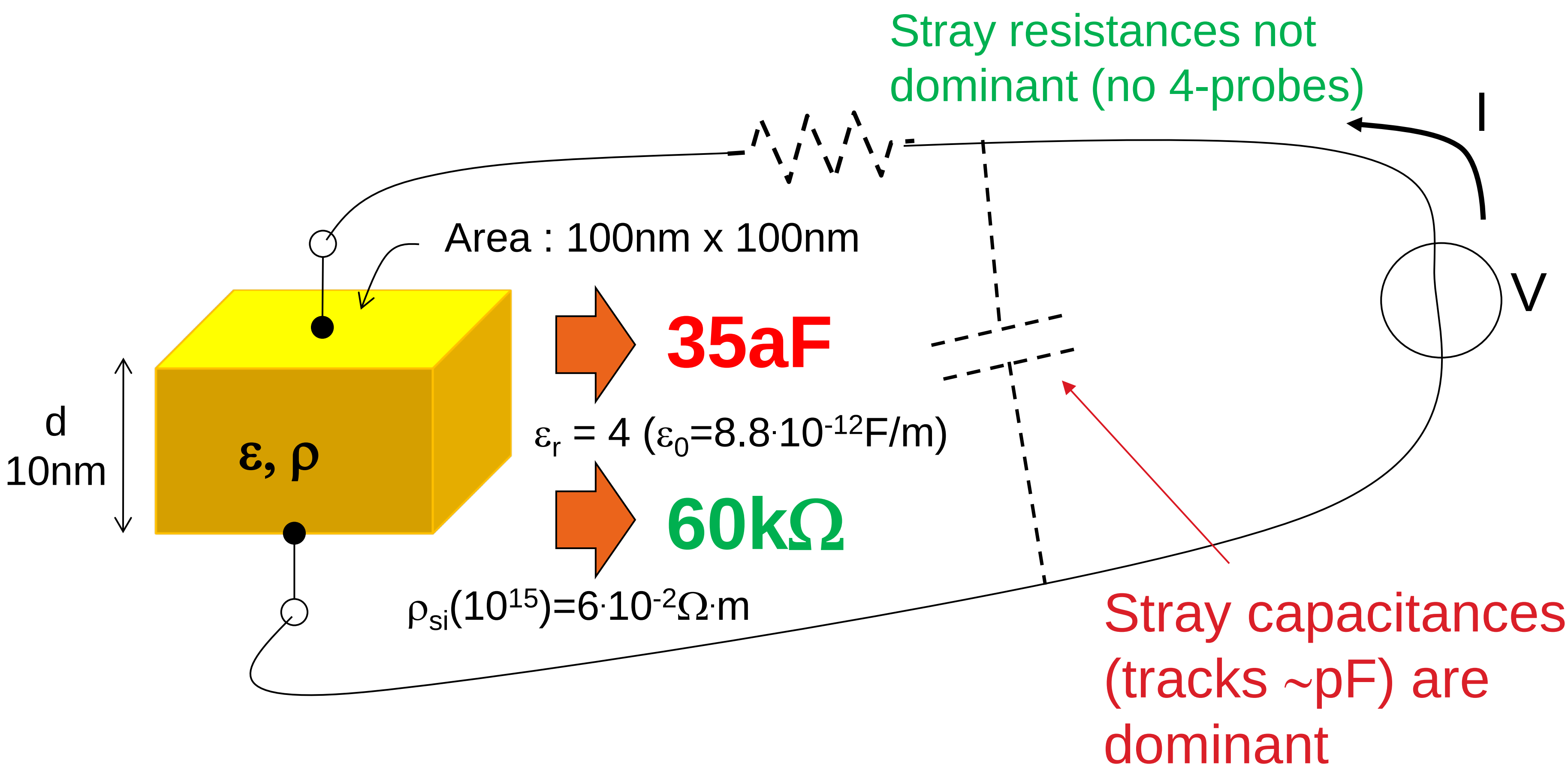
4 probes Impedance measurement



$$Z = \frac{V}{I} = Z_{DUT}$$

marco.sampietro@polimi.it

Impedance at the Nanoscale

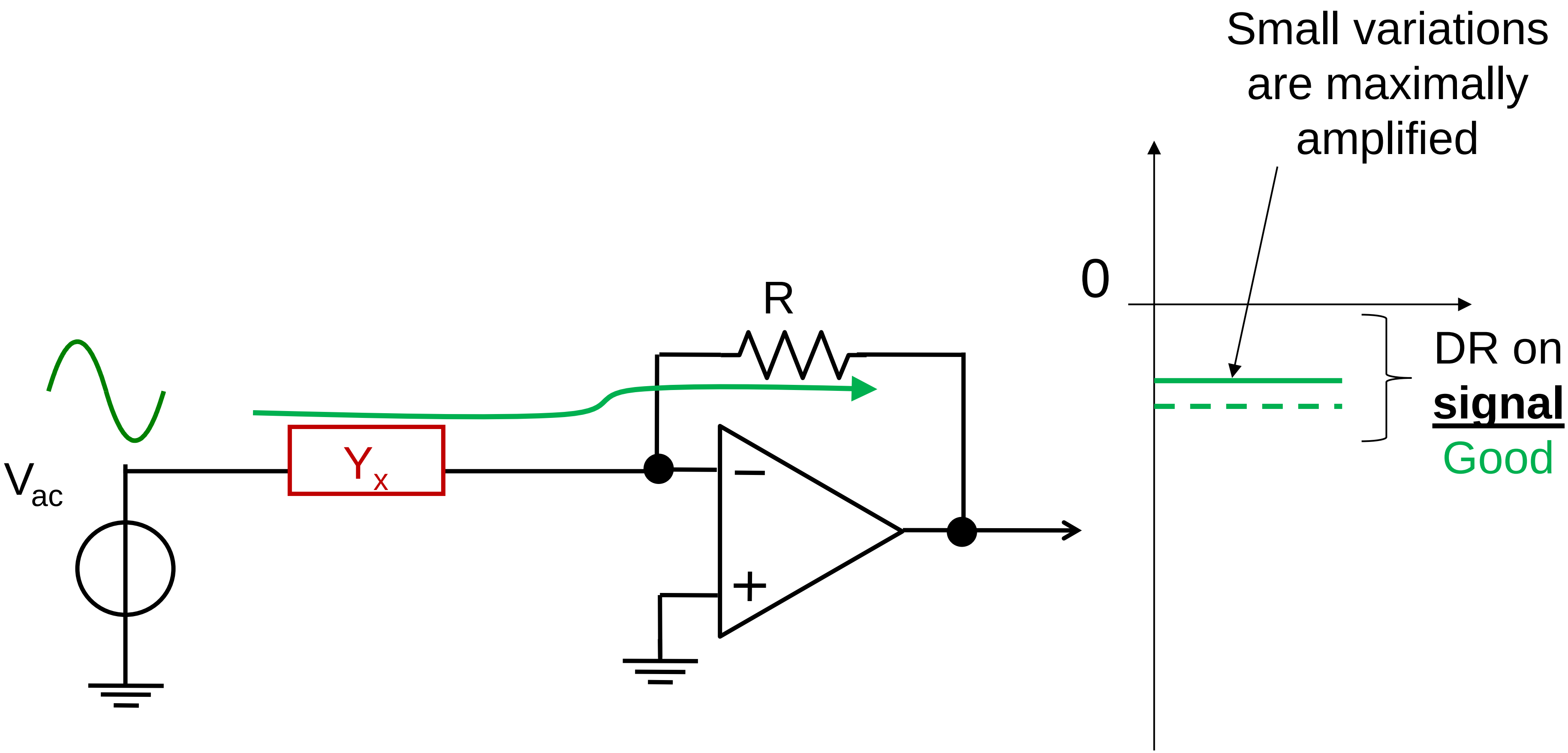


$$\tau = RC = \rho \frac{d}{Area} \cdot \varepsilon \frac{Area}{d}$$

$\tau = \rho \cdot \varepsilon = 2$ps ***independent*** of size

marco.sampietro@polimi.it

Problems given by strays (capacitance)



marco.sampietro@polimi.it

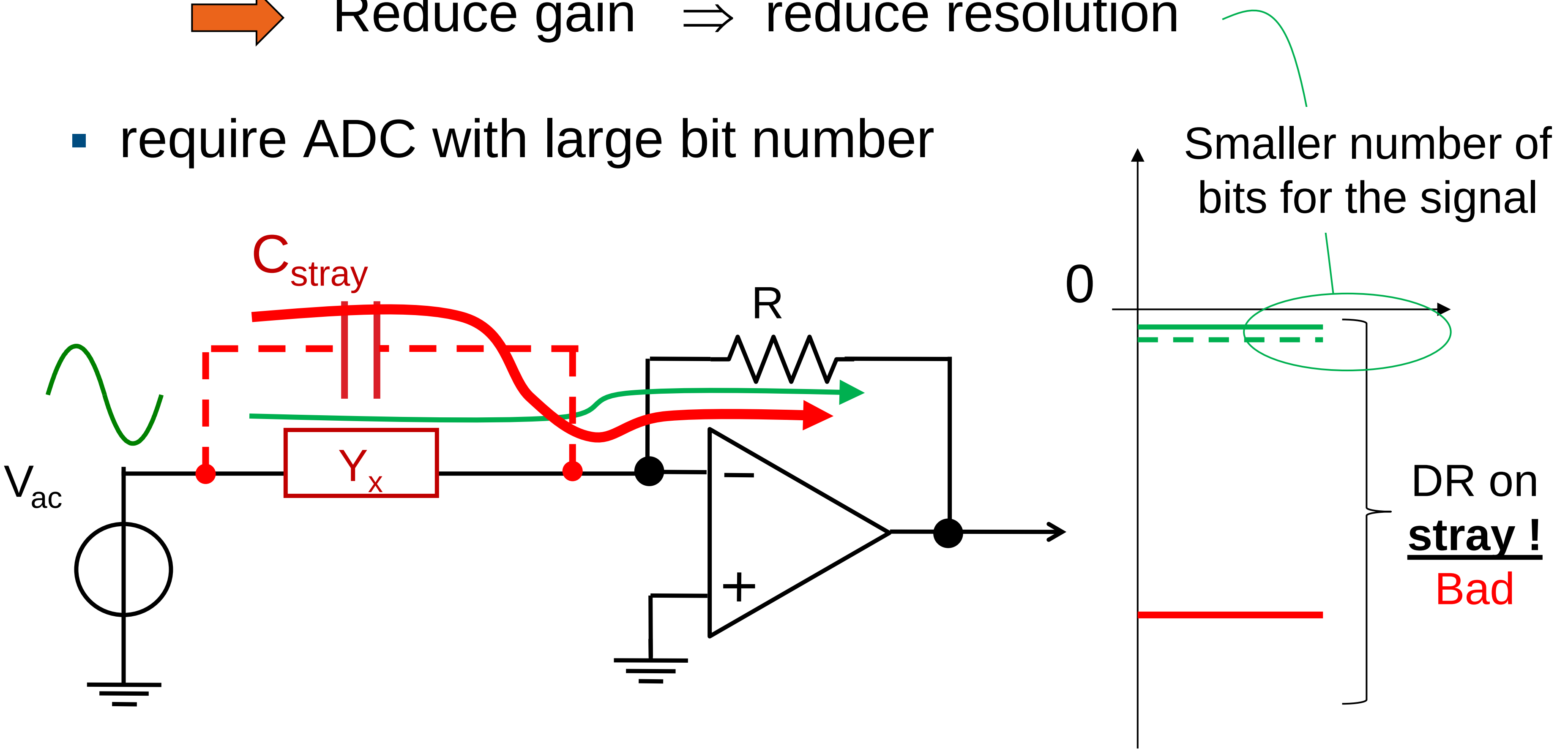
Reduction of sensitivity

A stray parallel capacitance C_{stray} may:

- saturate the front-end or gain stages

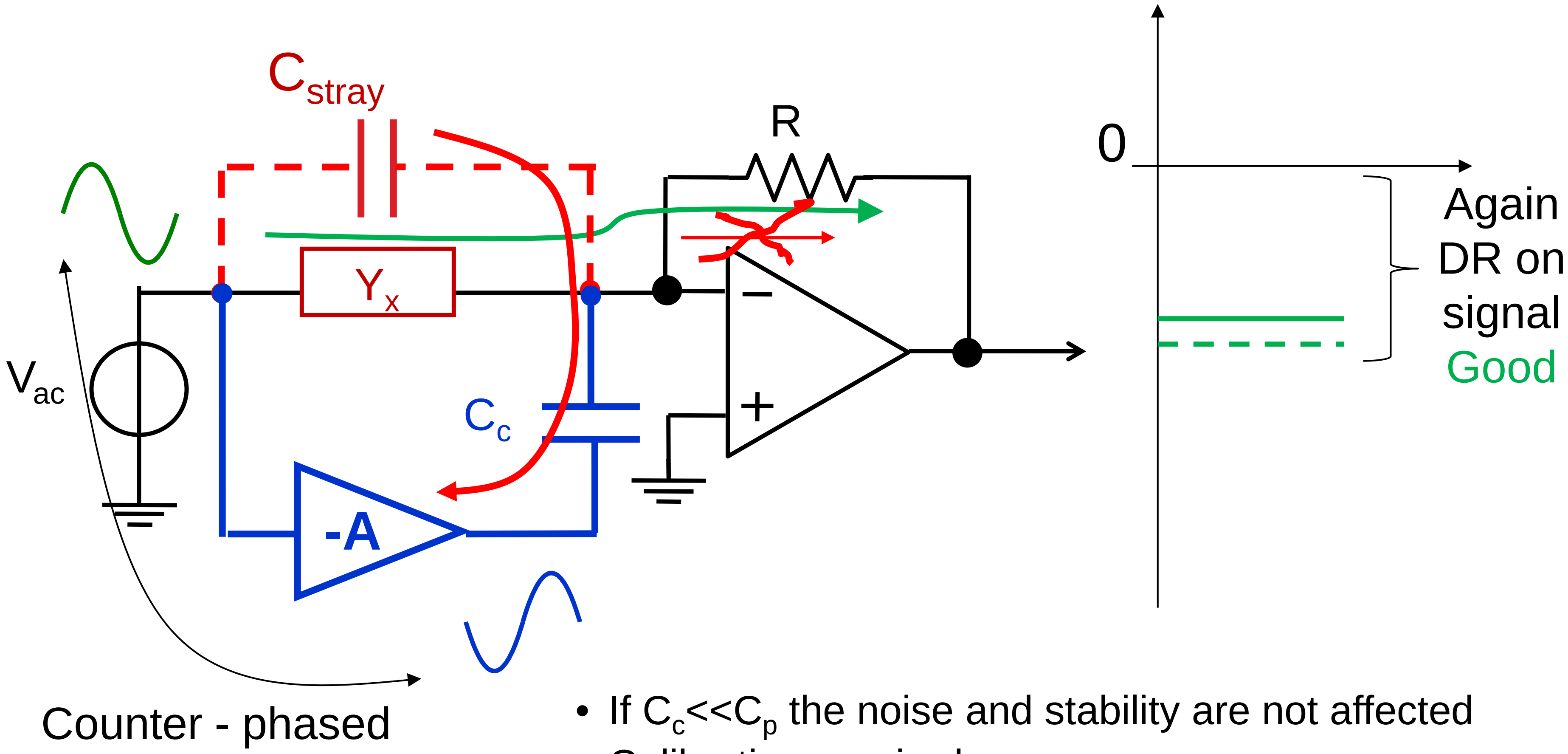
  ➡ Reduce gain $\Rightarrow$ reduce resolution

- require ADC with large bit number



Compensation in current sensing

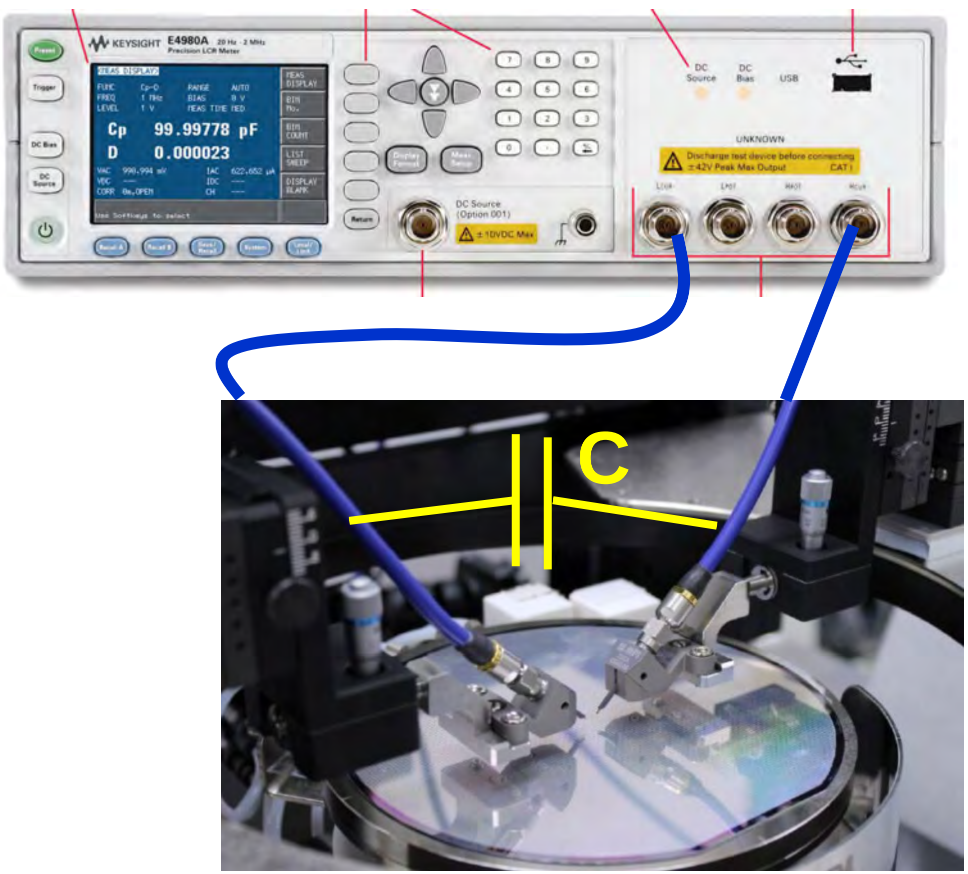
An **active capacitance compensation** can be useful:



- If C_c<<C_p the noise and stability are not affected
- Calibration required

marco.sampietro@polimi.it

Strays compensation in LCR meter (1

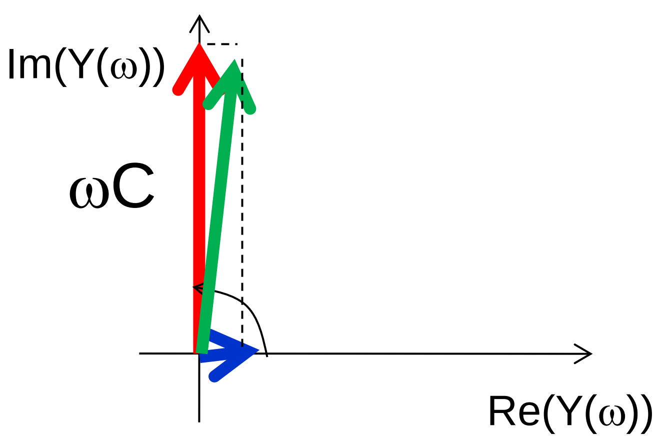


OPEN

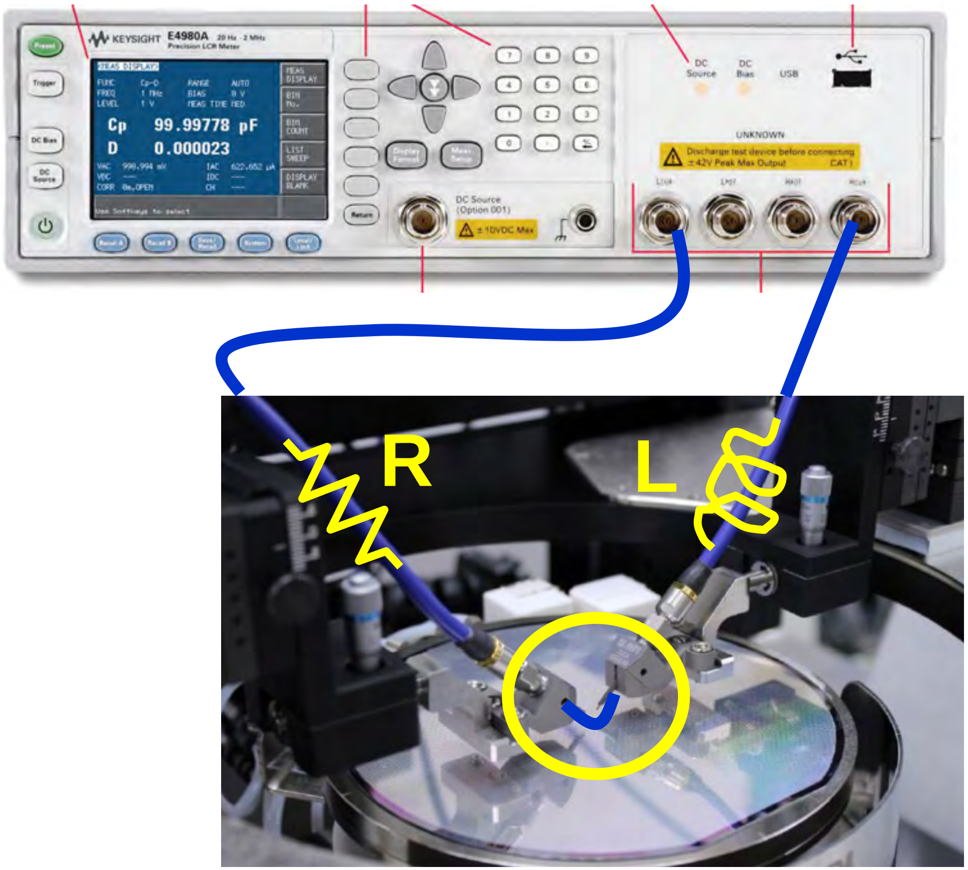
You Lift the probes (a little)

The instrument :

- Measures (the strays, mainly capacitance)
- Memorizes the values *Re* and *Im* at different f
- Correct the following meas. with these values



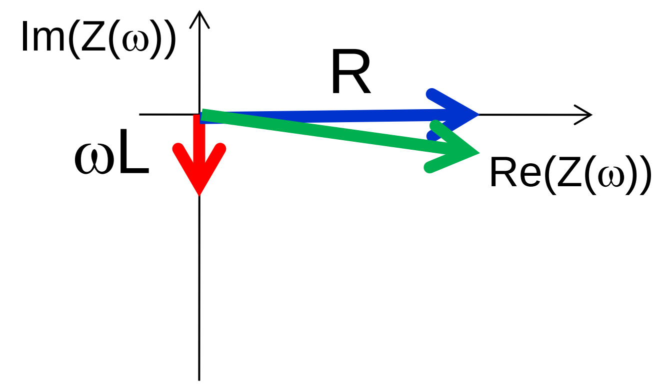
Strays compensation in LCR meter (2



SHORT
You Put probes in contacts

The instrument

- Measures (the strays, mainly resistance-inductance)
- Memorizes the values *Re* and *Im* at different f
- Correct the following meas. with these values



marco.sampietro@polimi.it

Strays compensation in LCR meter (3

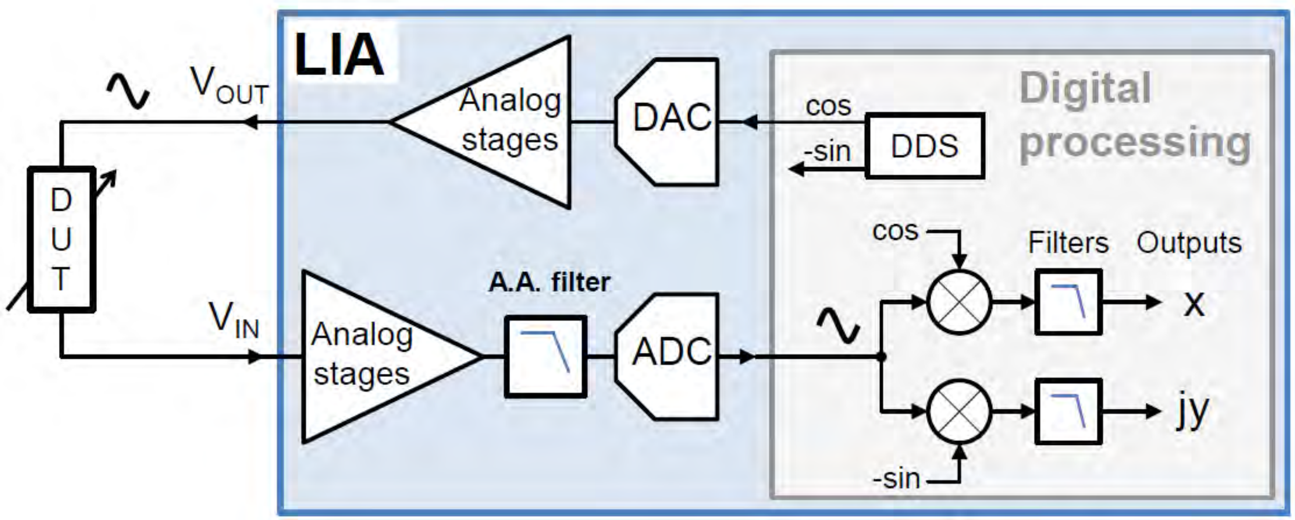
In addition USE 4 PROBES

marco.sampietro@polimi.it

Digital LOCK-IN amplifiers

Impedance spectroscopy with lock-in requires a separate measurement for each frequency → **long time**

Alternatives : Apply many-frequencies as stimulus and process in parallel;
Apply white noise at input and calculate the DFT of signals.



Next lesson by Giorgio Ferrari

chatGPT enquire

Myself : *How can I measure the impedance of a device ?*

chatGPT : Using an Impedance Analyzer

Using an LCR Meter

(Impedance Analyzer and LCR Meter: Lock-in architecture)

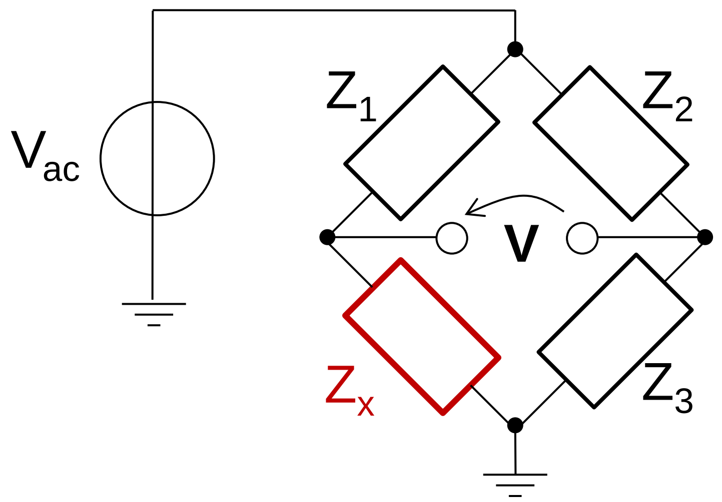
Bridge Circuit

Oscilloscope and Function Generator using the voltage-divider principle

Material follows for your convenience

marco.sampietro@polimi.it

Balancing Bridge: Working Principle



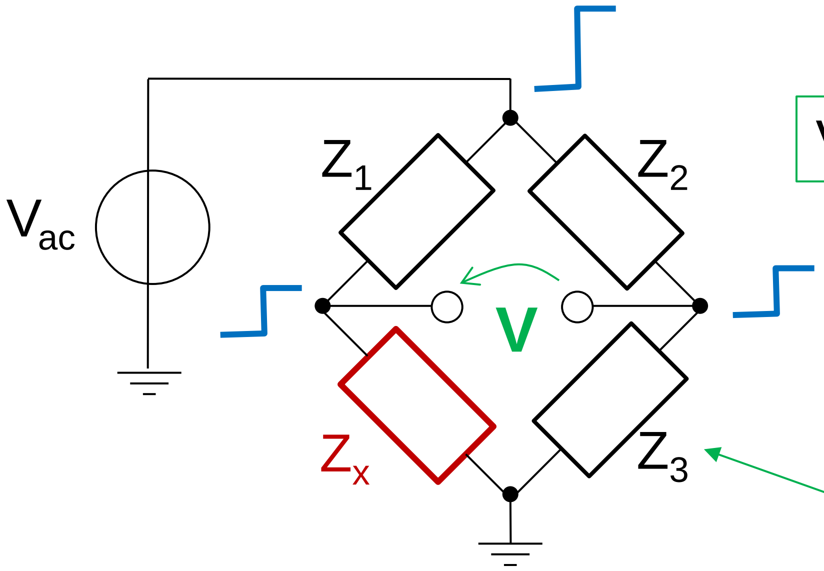
- Z_1, Z_2, Z_3 known and variable (switches)
- V_{ac} sinusoidal

$$\boldsymbol{V} = V_{ac}\left(\frac{\boldsymbol{Z_x}}{Z_1 + \boldsymbol{Z_x}} - \frac{Z_3}{Z_2 + Z_3}\right)$$

Balanced for **V = 0** ➡ $$\boldsymbol{Z_x} = Z_3 \frac{Z_1}{Z_2}$$

marco.sampietro@polimi.it

Bridge Pros and Cons



Pros:

Voltage reader operates always with V≈0V

Common mode rejection

Good accuracy (no active stages, depends on the accuracy of the reference impedances)

Cons:

- Requires several switches
- Slow balancing routine
- Not very convenient for spectroscopy

Examples of Commercial Instruments

GR 1650-A (1957) …fully manual

Andeen-Hagerling

AH 2700A

50Hz-20kHz

Capacitance Bridge

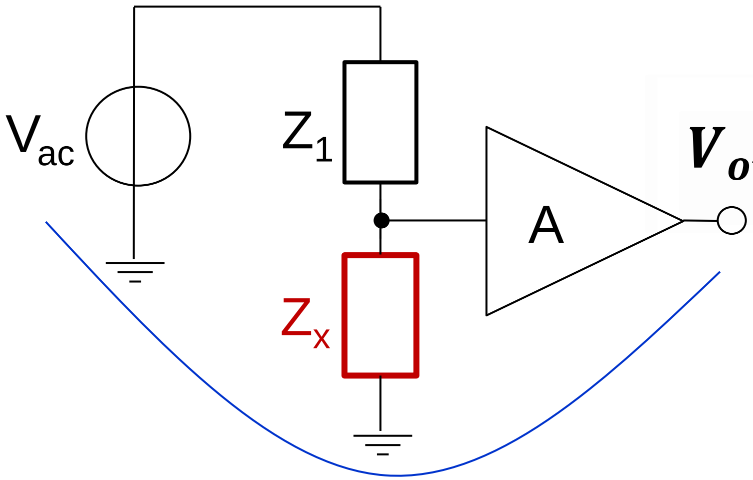
0.5aF, 1PΩ resolution

Ratiometric: Half Bridge

Ratiometric i.e. V_{out} depends on the impedance ratio

↓

Independent of the absolute value



$$V_{out} = V_{ac} \frac{Z_x}{Z_1 + Z_x} A$$

$$Z_x = Z_1 \frac{V_{out}}{AV_{ac} - V_{out}}$$

A phase sensitive detector is needed

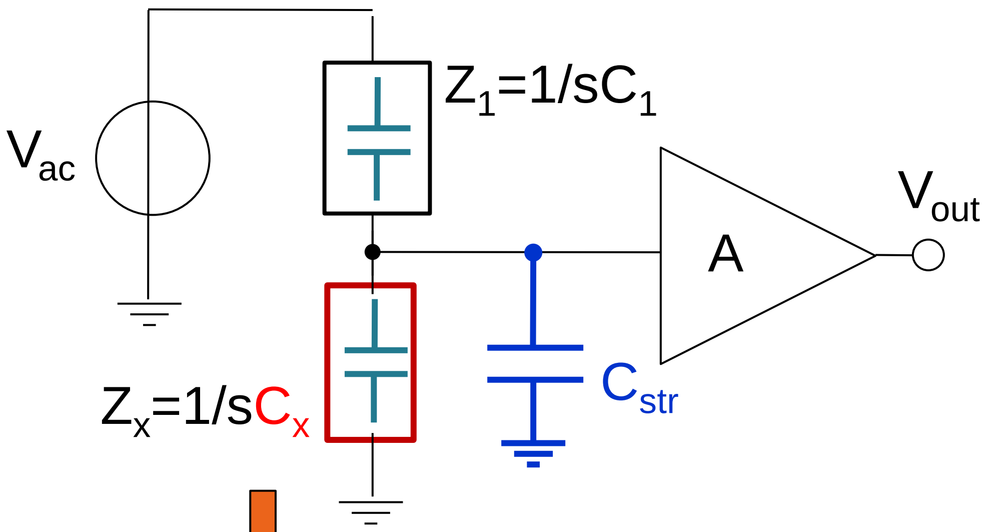
Z_1 has to be accurate (wide dynamic) :

- $Z_x >> Z_1$: $V_{out} \approx AV_{ac}$
- $Z_x << Z_1$: $V_{out} \approx 0$

→ $Z_1 \sim Z_x$

Difficult at the nanoscale

Capacitance detection: Effect of Cstray



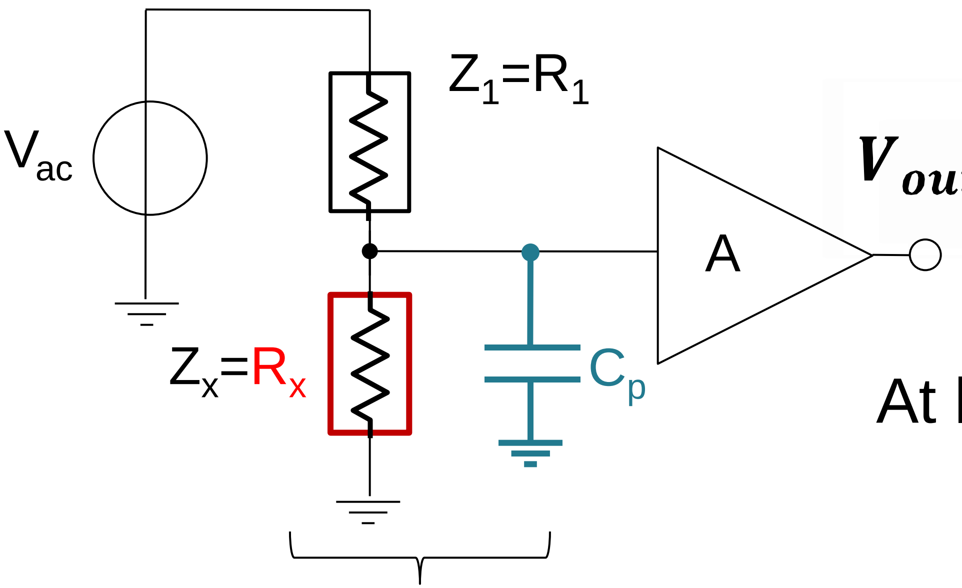
$$C_x = C_1 \frac{AV_{ac} - V_{out}}{V_{out}} - C_{str}$$

Reduces the accuracy !

DC bias of Z_x not defined

marco.sampietro@polimi.it

Resistance detection: Effect of C_{stray}



$$V_{out} = AV_{ac}\frac{R_x}{R_x + R_1}\frac{1}{1 + sC_pR_x||R_1}$$

At low frequencies ($\omega < 1/C_pR_x||R_1$)

$$R_x \cong R_1\frac{V_{out}}{AV_{ac} - V_{out}}$$

At high frequencies ($\omega > 1/C_pR_x||R_1$)

→ R_x shunted by C_p!

Example: a cube of intrinsic Si (~1kΩ cm), side = 50nm

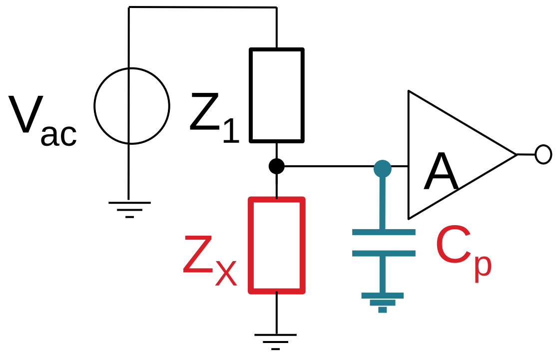
→ R_x = 200MΩ, cut-off frequency = 160Hz (C_p = 5pF)

marco.sampietro@polimi.it

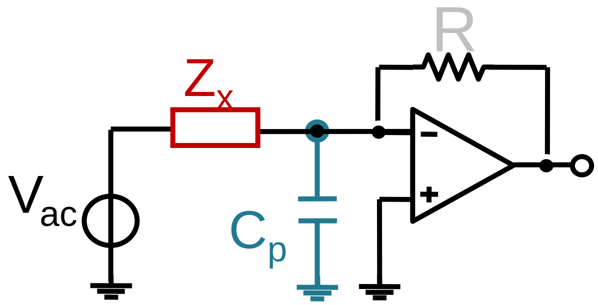
Comparison

Ratiometric:

- C_p limits bandwidth and accuracy
- No control of the voltage applied to Z_x
- Z_1 must match Z_x



Current sensing:



- Independent of C_p (wide-band opamp)
- Precise control of the voltage applied
- Need to access both terminals of Z_x
- Loop stability depends on Z_x (but at the nanoscale dominated by stray capacitance ≈ known)

In terms of resolution they are **equivalent**

marco.sampietro@polimi.it